





Center for Research in Thinking and Language

Department of Psychology

The Catholic University of America
Washington, D. C. 20064

Sources of difficulty in deducing instances from concepts

by

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more difficult to recognize than conjunctive ones. These same two subjects also showed the rapid improvement characteristic of insight. More gradual improvement and independence from logical properties of problems characterized the other subject's responding. For solution times, all subjects were slowest during the first two sessions. It was also found that the two "insight" achieving subjects were more influenced by logical properties of problems and paralleled rapid improvements in correctness with large drops in solution time. Findings were interpreted as evidence for two different strategies for solving instantiation problems -- memorizing correct answers versus discovering general problem solving principles.



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Sources of Difficulty in Deducing

Instances from Concepts

The present report is the fifth in a series concerned with instance deduction, the identification of valid instances of a specified set of concepts, performed by untrained adults. The first report (Ross, Locker, & DeLisi, 1974) was concerned with demonstrating the feasibility of studying instance deduction (concept instantiation). Both a production and a recognition method were administered and, similarly, production and recognition problems were also administered to other subjects with more traditional problems in which concepts were attained from instances. The logical rationale of instance-deduction problems was presented in a second technical report (Mott & Ross, 1975). In a third report (Ross & Gurney, 1975) results for subjects who performed a variety of instance deductions using deduction rules were contrasted with results obtained from subjects who obtained solutions without having any rules available. All problems were presented only at a single session.

In this report a case study approach is adopted of intensively studying a limited number of subjects performing moderately difficult instance-deduction problems over several sessions. Deduction rules were available and automatic feedback as to correctness was given. Unlike deductions performed for the earlier reports, the attempt is made to obtain data that pertain to at least four questions. Do the sources of problem difficulty remain the same over several sessions with repeated exposure to problems? Are there distinctive problem-solving styles? With no tuition other than repeated problem presentation is a high level of accuracy (at least 80 percent) achieved in recognizing correct instances? Additionally, what is the sensitivity of reaction time measures to different types of problems, both for initial attempts at solution and after problems have been performed in several sessions?

METHOD

Subjects.

Three Catholic University students participated as paid subjects at the rate of \$2.25 per hour. One graduate student male was age 22; a second male was an undergraduate of 20; the third subject was a 20-year-old undergraduate female. The subjects had no prior experimental experience or special training in logic.

Design and procedure.

The experiment consisted of eight sessions for each subject lasting approximately 30 minutes to one hour. Two different groups of problems were used, a relatively "easy" group consisting of 10 sets of concepts and a "hard" group consisting of 13 sets of concepts. Easy problems were presented in sessions one, three, and five while in the remaining five sessions hard problems were administered. (In the subsequent Results and Discussion sections, these hard sessions are referred to as sessions 1 through 5.) Only results from the sessions with hard problems will be analyzed and reported here. A more technical definition can be given of hard problems in that concepts for which instances were to be recognized were all either conjunctions of disjunctions or disjunctions of conjunctions (what we have elsewhere termed 2nd-order concepts).

Perhaps the most direct way in which problem specification can be illustrated is to list two of the problems that were administrated to the subjects on the CRT scope together with keyboard instructions. Concept sets remained displayed while instances were presented successively one at a time.

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Illustrative Problem No. 1.

CONCEPTS: LARGE OBJECT OR SINGLE RED OBJECT RED AND EITHER SMALL OR BORDERED OBJECT

DETERMINE WHETHER THE FOLLOWING INSTANCE IS A POSITIVE INSTANCE OF ALL THE LISTED CONCEPTS (ALL=PRESS A); NONE OF THE LISTED CONCEPTS (NONE=PRESS N); OR AT LEAST ONE BUT NOT ALL OF THE LISTED CONCEPTS (SOME=PRESS S).

PAIR OF SMALL BLACK UNBORDERED OBJECTS
PAIR OR SMALL OR BLACK OR UNBORDERED OBJECT
PAIR OF LARGE BLACK BORDERED OBJECTS
PAIR OR LARGE OR PLACK OR BORDERED OBJECT
PAIR OF LARGE RED BORDERED OBJECTS
SINGLE OR SMALL OR BLACK OR UNBORDERED OBJECT
SINGLE SMALL RED UNBORDERED OBJECT
SINGLE OR SMALL OR RED OR UNBORDERED OBJECT
SINGLE LARGE RED BORDERED OBJECT
SINGLE OR LARGE OR RED OR BORDERED OBJECT

Illustrative Problem No. 2.

CONCEPTS: SINGLE SMALL OBJECT OR BORDERED OBJECT
SINGLE LARGE OBJECT OR PAIR OF BORDERED OBJECTS
SINGLE SMALL OBJECT OR LARGE BORDERED OBJECT

DETERMINE WHETHER THE FOLLOWING INSTANCE IS A POSITIVE INSTANCE OF ALL THE LISTED CONCEPTS (ALL=PRESS A); NONE OF THE LISTED CONCEPTS (NONE=PRESS A); OR AT LEAST ONE BUT NOT ALL OF THE LISTED CONCEPTS (SOME=PRESS S).

PAIR OF SMALL BLACK UNBORDERED OBJECTS
PAIR OR SMALL OR BLACK OR UNBORDERED OBJECT.
PAIR OF LARGE BLACK BORDERED OBJECTS
PAIR OR LARGE OR BLACK OR BORDERED OBJECT
PAIR OF LARGE RED BORDERED OBJECTS
SINGLE OR SMALL OR BLACK OR UNBORDERED OBJECT
SINGLE SMALL OR RED OR UNBORDERED OBJECT
SINGLE OR SMALL OR RED OR UNBORDERED OBJECT
SINGLE LARGE RED BORDERED OBJECT
SINGLE OR LARGE OR RED OR BORDERED OBJECT

It can be noted in the problem illustrations that there were the same set of 10 instances for each problem; this was true also of the other 11 problems, making a total of 130 instance recognition decisions in all to be performed during each session. Owing to an experimenter error, a total of 20 fewer hard problems were administered to one subject. It was considered likely that performing this number of recognitions each session mitigated against the possibility of memorizing results rather than attempting logical deduction. If straight memorization were attempted, the fact that the same instances were repeated each time with a different set of concepts should, at the least, lead to considerable interference.

Also in the interest of balancing problem types and making valid intersession comparisons possible, the same problems were administered for each "hard" session. Recall that different "easy" problems (i.e., concept sets with some first-order as well as second-order concepts) were administered in the first, third, and fifth sessions. Additionally, in the sixth session the hard problems were administered in the reverse order of that given in the second and fourth sessions. Finally, in sessions seven and eight, different random problem orders were administered for each subject at each session. Thus direct memorization of a particular instance with a specific set of concepts would have been very difficult.

Subjects were given a printed set of four deduction rules at the beginning of each problem recognition session. The rules, that could be referred to by the subjects at any time, were the same as those administered previously (Ross & Gurney, 1975). The deduction rules are here printed in the Appendix. An initial reading of the rules and answering of any questions was performed before the subject entered the experimental booth containing the CRT scope. No practice problems were given. Subjects proceeded at their own speed. After the subject pressed the "all," "none," or "some" key, "correct" or "incorrect" appeared on the scope to give feedback as to correctness. The subject then pressed a button to obtain the next instance presentation. The subject was given a break of 30 seconds after each 10 instance recognitions and an approximately a five minute break halfway through each session. One subject refused any break during the last few sessions.

RESULTS

Findings of this study are presented in two parts. These parts present analyses of the variance of correct response proportions and solution times, respectively.

Correct Response Proportions

Table 1 presents results of analyzing the variance of correct response proportions. (See next page for Table 1)

TABLE I

Summary Table for ANOVA of Correct Response Proportions

Source	M.S.	df	F	\mathcal{N}^{2} a
Session (A	2.4100	4	5.5466 *	.086
Problem Type (H	3) .0174	2	. 3551	
Answer Type (C	2) 4.8002	1	2.0588	
Subjects (S	26.4873	2	179.3318 ***	. 471
АВ	.2638	. 8	2.5513	
A C	1.0247	4	1.5709	
A S	. 4345	8	2.9418 **	.031
ВС	. 7388	2	1.1410	
B S	. 0490	4	.3318	
c s	2.3316	2	15.7861 ***	. 041
ABC	. 1715	. 8	. 1516	
ABS	. 1034	16	. 7001	
ACS	. 6523	8	4.4164 ***	. 046
всѕ	. 6475	4	4.3839 **	. 023
ABCS	1.1310	16	7.6574 ***	. 161
Replications	. 1477	1860		

^{*} P<,05

^{**} P < .005

^{***} P < .001

a \mathcal{N}^2 represents the proportion of variation in correct response proportions that is attributed to the corresponding source.

Table 2 presents proportions of correct responses on which the analysis of Table 1 is based.

(Table 2 is on page 10)

The analysis was for proportional but unequal cell frequencies (q.v., Winer, 1971).

The significant effect of Sessions is clear from inspection of Table 2, or more directly from consideration of the overall means for sessions. These means are .645, .692, .733, .803, and .837 for sessions 1 to 5 respectively. They show, of course, the improvement over time that is to be expected in tasks of this nature.

By far the most important source of variation is that of Subjects. This source accounts for almost half (.471) of all the variance for correct response proportions. Subject 1's mean was .531; Subject 2's was .935; Subject 3's was .760.

In addition to these two main effects, Sessions and Subjects, five interactions were significant. Table 3 presents the means on which the significant

(Table 3 is on page 11)

Sessions X Subjects interaction is based. The primary source of the Sessions X Subjects interation is the fact that (allowing for the adjustment in session 1³) Subject 1 improves slowly but continually over Sessions,

- 1. This analysis entailed "creating" twenty response for Subject 1 during session 1 since Subjects 2 and 3 each responded to 20 problems more than Subject 1. These responses were created by assuming that Subject 1's responses to the missing problems could be assigned values equal to the grand mean of correct response proportions for all three subjects across all conditions. This is a conservative assumption for tests of main effects. Of the data analyzed in Table 1, the 20 created responses constitute about one percent.
- 2. In part, the Sessions X Subjects interaction is an artifact of the analysis used. Were the effects of Sessions and Subjects additive, one would expect Subject 1's first session mean to be closer to .4 (its empirical value) than to .5 (its adjusted value).
- 3. See footnote 1.

TABLE 2

Correct Response Proportions of Three Subjects for a Series of Session Instantiation Problems

				S	ESS	ION		
Subject	Problem	Answer						
	Type	Type	1	2	3	4	5	
			h					-
1	Da	Da	. 48 b		. 65	. 85	. 80	
		Ca	. 48 b		. 40	. 35	. 45	
	Ca	D	. 50	.40	. 30	. 60	. 70	
		С	.40	. 40	.40	. 70	. 90	
	Мa	D	.50°	.51	.51	. 46	.51	
		С	.55 d	.51	. 63	.51	. 71	
								-
2	D	D	.55	. 95	1.00	. 95	1.00	
		C	1.00	1.00	1.00	1.00	1.00	
	C	D	. 70	. 60	. 90	. 90	. 90	
		C	1.00	1.00	1.00	1.00	1.00	
	M	D	.54	1.00	1.00	1.00	1.00	
		С	. 91	. 97	1.00	1.00	. 97	
								-
3	D	D	. 45	. 45	.50	1.00	. 95	
		С	. 80	. 75	. 80	. 95	. 95	
	C	D	.60	. 70	. 70	. 80	. 70	
		C	1.00	. 70	.70	. 80	1.00	
	M	D	.51	. 49	. 46	. 83	. 83	
		С	. 83	. 94	. 94	. 86	. 88	

- a D = disjunctive C = conjunctive
 - M = mixed
- b This is adjusted to proportionalize cell frequencies. Not adjusted, this proportion is . 30
- c Not adjusted, the proportion is . 40
- d Not adjusted, the proportion is . 45

TABLE 3

Correct Response Proportions for Sessions X Subjects

		SE	SSION	1	
C 1:	1	2	3	4	5
Subject 1	.50 ^a	. 44	. 52	. 55	. 64
2	. 76	. 95	. 99	. 98	. 98
3	.68	. 68	. 68	. 88	. 88

a See footnotes b, c, d to Table 2.
Not adjusted for the ANOVA, this proportion would be .40

while Subjects 2 and 3 improve in a more step function manner. Subject 2 shows a 19 percent improvement from Sessions 1 to Session 2. Subject 3 shows a similar jump between Sessions 3 and 4 of 20 percent.

Means for the Solution Type X Subjects interaction are in Table 4. The Solution Type X Subjects interaction clearly results from Subject 2 and even more from Subject 3 finding problems with disjunctive answers to

(Table 4 is on page 12)

be more difficult than those having conjunctive answers. Subject 1 performed equally poorly with both types of problem.

TABLE 4

Correct Response Proportions for Answer Type
X Subjects

	Answ	er Type
Subject	Disjunctive	Conjunctive
l	. 54 ^a	. 52 a
2	.89	. 98
3	. 65	. 87
	· · · · · · ·	

a See footnotes b, c, d, to Table 2.

Means for the remaining three significant interactions are readily determinable from Table 2. In addition to the previously presented results, the data in Table 2 give greater detail about the step function character of Subject 2's and Subject 3's improvement. If we take any improvement for adjacent sessions in excess of 25 percent as a "step," there are five steps in Table 2. Between Sessions 1 and 2, Subject 2 improves in solving disjunctive problems having disjunctive answers and mixed problems having disjunctive answers by 40 percent and 46 percent respectively.

Between Sessions 2 and 3, he improves in solving conjunctive problems having disjunctive answers by 30 percent. Like Subject 2 during the first two Sessions, Subject 3 between Sessions 3 and 4 improved in solving disjunctive-disjunctive and mixed-disjunctive problems. Her respective improvements were 50 percent and 37 percent. There are no such improvement steps in Subject 1's data.

Solution times

Table 5 summarizes an analysis of the variance of solution times. 4 Means

(Table 5 is on page 14)

for the analysis are presented in Table 6.

(Table 6 is on page 15)

The significant main effect for Sessions is reflected in successive means of 17.7, 13.3, 9.9, 10.4, and 9.3 seconds. When sessions are considered in their order of administration, by far the largest changes occur between Sessions 1 and 2, and Sessions 2 and 3. The Sessions effect accounts for 61.5 percent of the variance.

The main effect for Solution Type is based on means of 12.6 and 11.6 seconds for disjunctive and conjunctive answers, respectively. This result indicates slightly faster solution times for the conjunctive solutions, as we would expect if solution time increased with problem difficulty. The small magnitude of this effect is reflected in the fact that it accounts for only 1.4 percent of mean solution time variation.

^{4.} An adjustment was made for Subject 1's first session data in order to make the unequal frequencies proportional. This adjustment was the same substitution of a grand mean as used for the analysis of correct response proportions.

TABLE 5
Summary Table for ANOVA of Solution Times

Source	M.S.	df	F	n ^{2 a}
Sessions (A)	4703. 946	4	10.225 ***	. 615
Problem Type (B)	1006.834	2	3.541	
Solution Type (C)	439. 731	1	24.651 *	. 014
Subjects (S)	430.286	2	17.416 ****	.028
АВ	81.665	8	1.319	
A C	99. 734	4	1.735	
A S	460.022	8	18.619 ****	. 120
ВС	1. 743	2	.016	
ВЅ	284. 329	4	11.508 ****	.037
СS	17.838	2	. 722	
ABC	29.203	8	1.081	
ABS	61.914	16	2.506 **	. 032
ACS	57.484	8	2.327 *	.015
BCS	105.667	4	4.277 ***	.014
ABCS	27.026	16	1.094	
Replications	24.707	1860		

^{*} p < .05
** p < .01
*** p < .005
*** p < .005

^{****} p . . 001

a PRepresents the proportion of mean solution time variance attributable to the corresponding source.

TABLE 6

Mean Solution Times (in seconds) of Three Subjects for a Series of Instantiation Problems

Subject	Problem	Answer		SE	SSI	ON	
	Туре	Туре	1	2	3	4	5
1	Da	Da	13.2 b	12.6	12.4	7.8	10.0
		Ca	11.6 ^c	12.0	11.0	9.0	9.3
	Ca	D	17.0	18.0	11.6	10.2	11.3
		C	10.3	12.9	10.8	9.6	9.9
	Мa	D	14.1 d	12.7	10.1	10.0	10.6
		C	16. 7 ^e	12.3	9.0	9.6	10.3
2	D	D	19.3	13.4	7.8	8.8	7.6
		C	15.0	12.4	7.0	9.0	6.8
	C	D	17.1	18.1	7.4	8.9	7.7
		C	17.5	18.5	9.8	11.5	10.2
	M	D	26.8	16.2	10.2	13.3	9.5
		C	20.3	16.6	10.7	13.5	7.0
3	D	D	19.4	11.4	8.2	8.3	8.2
		C	15.0	11.8	8.2	9.8	8.0
	С	D	19.0	10.1	6.7	8.9	7.7
		C	14.7	9.0	5.7	6.9	7.1
	M	D	20.1	11.7	13.4	12.6	11.3
		С	17. 9	11.9	10.9	10.6	10.4

a D = disjunctive
 C = conjunctive
 M = mixed
b)
c)
d) Not adjusted means are 13.6, 11.4, 14.5, 17.5, respectively.
e)

While significant at $\underline{p} < .001$, the Subjects effect is much less important for solution time means than it is for correct response proportions. In Table 5 it account for only 2.8 percent of the variation while in Table 1 it accounts for 47.1 percent. The means for this effect are 11.4, 13.0, and 11.9 seconds for Subjects 1, 2, and 3, respectively. It is noteworthy that these means are ordered in the same way as the Subject correct response proportions were ordered. The longer a subject's average solution time, the larger the subject's average proportion of correct responses.

Next to Sessions, the largest contribution to the variation of mean solution times is given by the effect of Sessions X Subjects. Means for this interaction are in Table 7.

(Table 7 is on page 17)

Considering each subject's sequence over sessions, it becomes clear from Table 7 that for all subjects, the longest average solution time was for Session 1, and the second largest was for Session 2. For Subject 1, the last three session means are ranked Session 3 longest, then Session 5, then Session 4. For Subjects 2 and 3, the ranking is Session 4, Session 3, Session 5. These means for the last three Session are sufficiently homogeneous that they may represent an equilibrium for the variable mean solution time.

Table 8 presents means for the Problem Type X Subjects interaction. The interaction can be seen by ordering each Subject's means for Problem

(Table 8 is on page 18)

Types. For Subject 1, disjunctive problem types were responded to more rapidly than were mixed types, which were themselves responded to more rapidly than conjunctive types. A similar ordering for Subject 2 ranks the types as disjunctive (shortest mean solution time), conjunctive, mixed. Subject 3's ranking is conjunctive, disjunctive, mixed.

Recall that for correct response proportions presented in Table 4, the correctness of Subject 1's responding was less influenced by answer characteristics than was true for Subject 2 or 3. A complementary result is shown again for solution time means in Table 8. Across the three Problem Type means, Subject 1's range is 1.3; for Subject 2, the range is 3.7; for Subject 3, the range is 3.5. Subjects 2 and 3 thus had solution times more dependent on Problem Type than did Subject 1. Between Subjects 2 and 3, a difference emerges in solution times for disjunctive

TABLE 7

Mean Solution Times (in seconds) for Sessions

		X Subjects			
		s	ESSI	0 N	
Subject	1	2	3	4	5
1	14.2ª	12.9	10.5	9.4	10.2
2	20.6	15.6	9.2	11.5	8.1
3	18.1	11.4	10.0	10.2	9.5

a See footnotes b, c, d, e to Table 6.

Mean Solution Times (in seconds) for Problem Type X Subjects

TABLE 8

	P	roblem Ty	p e
Subject	Disjunctive	Conjunctive	Mixed
1	10.9ª	12.2	11.5 b
2	10.7	12.7	14.4
3	10.8	9.6	13.1

a See footnotes b, c, to Table 6.b See footnotes d, e, to Table 6.

and conjunctive problem types. Subject 2 responded more quickly to disjunctive types while Subject 3 responded more quickly to conjunctive types.

It is of interest to note that for Subject 2, the session-to-session changes that we have called "steps" are mirrored in his mean solution times. Recall that for correct response proportions, three improvement steps were found. Between Sessions 1 and 2, large increases in correct response proportions occurred for disjunctive-disjunctive problems and for mixed-disjunctive problems. For these same two sessions, Subject 2's mean solution times show the largest drops (5.9 and 10.6 seconds, respectively). Similarly, between sessions 2 and 3, a step was found for conjunctive-disjunctive problems. Subject 2's mean solution time for this same transition shows the largest between session drop (10.7 seconds). For Subject 3 whose two steps for correct response proportions were between Sessions 3 and 4, the corresponding changes in mean solution times rank first and third rather than first and second. Moreover, the third ranking change (for disjunctive-disjunctive problems) is a slight increase (.1 second) rather than a drop.

Finally, while the interaction of Answer Type X Subjects was not significant, the trend of the data is important for the subsequent Discussion section.

Table 9 presents mean solution times for this interaction. The important

(Table 9 is on page 20)

property of Table 9 is the result that Subject 1's range for mean times is only about half as large as similar ranges for Subjects 2 and 3 (.6 compared to 1.0 for Subject 2 and 1.3 for Subject 3).

TABLE 9

Mean Solution Times (in seconds)
for Answer Type X Subjects

	Answ	er Type
Subject	Disjunctive	Conjunctive
1	11.7ª	11.1ª
2	13.5	12.5
3	12.5	11.2

a See footnotes b, c, d, e in Table 6.

DISCUSSION

This study will be discussed in three parts. The first two parts parallel the two sections of the Results. The third part is more general.

Correct response proportions

Most of the variation in correct response proportions can be attributed to

- (a) relatively stable individual differences and
- (b) differences between levels at which a subject learns.

The individual differences require little comment other than repeating that the effect of Subjects alone accounted for over 47 percent of the variance in correct response proportions.

With respect to the second consideration, the analysis of variance together with the means in Table 2 suggest that Subject 1 was engaged in an activity like rote learning. Subjects 2 and 3, on the other hand, seem to have been discovering how one solves problems possessing disjunctive answers.

One expects a subject to have greater difficulty in recognizing a disjunctive instance than recognizing a conjunctive one. While this was, in fact, true for Subjects 2 and 3, it was not true for Subject 1. Subject 1 also showed no dramatic improvements between sessions as did Subjects 2 and 3. Yet, Subject 1 did improve over sessions. This improvement together with a relative independence from characteristics of the answers, suggests that the subject learned the correct responses as a result of feedback provided rather than by discovering a general principle for solving problems.

Subjects 2 and 3, on the other hand, replicated earlier research (Mott, Gurney, and Ross, submitted; Ross & Gurney, 1975) in having greater difficulty with disjunctive instances. Both of these subjects showed dramatic improvements between adjacent sessions for solving disjunctive problems. Moreover, improvements of this magnitude are found only when the answer is a disjunction. In contrast to Subject 1's seemingly rote process, Subjects 2 and 3 have "insights" (i.e., relatively rapid improvements) with respect to disjunction. This is most dramatically shown in Subject 3's session means (see Table 3). For each of the first three sessions, the mean is .68. For each of the remaining two, the mean is .88. If one asks what produced this change between sessions 3 and 4, reference to Table 2 shows that the largest contributions to the change are made by rapid improvement for disjunctive—disjunctive and mixed—disjunctive problems. This is in line with expectation if one supposes that Subject 3 discovered something fundamental, or at least widely applicable, with respect to disjunction.

Solution times

The overwhelming fact about the solution time data is that most of the variability is a main effect of Sessions. Session I had the longest time, Session 2, the second longest, and Sessions 3, 4, and 5 had similarly short solution times.

In general, Subject 1's solution times are more homogeneous than are those of Subjects 2 and 3. This is seen in Table 6. For the special comparison of Problem Types, Subject 1's solution times were less influenced by Problem Type than was true for Subjects 2 and 3. This complements the earlier finding that Subject 1's correctness had little to do with answer type. For one dependent variable, then, Subject 1 is less dependent on the answer type, while for the other dependent variable Subject 1 is less dependent on the problem type. These findings buttress the thesis that Subject 1 learned correct responses rather than how to work with logical properties of the problems. Similarly, while not statistically significant, the interaction presented in Table 9 reveals a trend for Answer Type to be less predictive of Subject 1's solution times as was also true for Subject 1's correct response proportions (see Table 4).

Subjects 2 and 3 showed drops in solution times paralleling dramatic improvements in correct response proportions. This lends support to the assertion that Subjects 2 and 3 show more insight than rote learning. For these two subjects, the speed of response depends on logical properties of both the presented concepts and the supplied answers. For Subject 1, on the other hand, only Answer Type made a difference in solution times, and to a lesser extent than for Subjects 2 and 3.

General

In summary, findings of the present study suggest that correct responding is influenced both by overall Subject differences and by whether the Subject works at solving concept-to-instance problems or uses rote learning. The overall subject difference variable may be as specific as the "ability to solve instantiation problems" or more comprehensive like an "intelligence" factor. It is not possible to determine this question of specificity with the present data.

The two approaches subjects took to their task led to two different patterns of correct response proportions. Subjects who could be theorized as using a more abstract, problem solving approach, showed dramatic improvements between adjacent sessions for problems having disjunctive answers.

Moreover, correctness of responses by the "Abstract" subjects depended on logical properties of answer types. It was additionally found that the

two subjects labeled "Abstract" were correct more often than the other subject.

The approach taken by the other subject was more concrete. This approach may be theorized as involving the conversion of instantiation problems into paired-associate learning problems. The instantiation itself is taken as a stimulus while the correct response (yes, no, some) is taken as the response to be learned. Such a "Concrete" subject shows a more gradual improvement in correct responding. Also, by not attending to the logical form of a problem, correct response proportions do not depend on the logical form of answer supplied.

To describe solution-time results, two variables are needed. The first is the Concrete-Abstract distinction just described for correct response proportions. Mean solution times for the "Abstract" subjects were again more strongly dependent on logical properties of concepts than on answers. For the "Concrete" subject, neither variable was as influential. Again these results support the difinitions of these two approaches. The concrete learning of correct responses in contrast to the abstract discovery of general principles for analytical problem solving is rather independent of the logical properties of the problem.

Present findings also suggest that (a) concrete learning has shorter solution times during early sessions, but in later sessions there is little difference between concrete and abstract solution times (see Table 7). Additionally, the data in Tables 2 and 6 suggest that the dramatic improvements or "steps" shown by abstract subjects are paralleled by drops in solution-time means.

The other variable required to describe the solution-time data is a Sessions effect that holds for both concrete and abstract subjects. This effect is one of decreasing times at the beginning of the series of sessions followed by greater stability for later sessions. This Sessions effect alone accounted for 61.5 percent of solution-time variability.

The findings of this study have importantly extended earlier instantiation research. In addition to the earlier noted replication of previous findings for correct response proportions, the present study looked at solution times over extended sequences of instantiation problems. This expansion of the design has enabled us to isolate the Concrete-Abstract distinction in a way not previously possible. We have also pointed out how solution time and correctness simultaneously reflect improved insight achieved by subjects who have obtained increasing problem familiarity.

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APPENDIX

Subjects Rules for Performing Instance Deductions

In the following problems you will be given a concept or concepts and lists of instances of concepts. In each problem you are to determine whether each listed instance is a correct instance of the given concept or concepts. If it is, write yes; otherwise write no.

Concepts and instances are specified by the following four dimensions:

Quantity:

Single-Pair

Size:

Large-Small

Color:

Red-Black

Perimeter:

Bordered-Unbordered

Instances and concepts are of two kinds. For example, "single small black bordered object" is an and concept, while "single small object or black bordered object" is an or concept. Observe that when the word "or" appears in a concept or an instance, it is used inclusively to mean "and/or." Thus the concept "large or red object" refers inclusively to either a large object, a red object, or a large red object.

And and or instances have different relations to the given concepts in order to be correct instances. There are four (4) rules to apply.

First, consider an and concept with and instances

Concept: S:

Small Red object

a correct instance:

pair of small red bordered

objects

an incorrect instance:

pair of small black unbordered

objects

Explanation:

The and concept must be completely matched by the

and instance for the instance to be correct.

Second, consider two or concepts with and instances

Concepts:

Single object or Bordered object

Pair of objects or Black Unbordered object

a correct instance for both

concepts:

single large black unbordered

object

an incorrect instance for

both concepts:

single large red unbordered

object

(Appendix, cont.)

Explanation:

Only one part (disjunct) of each or concept need be completely matched by the and instance for the instance to be correct. Thus in the above correct instance "single" is a sufficient match for the first concept and "black unbordered" for the second concept. The incorrect instance does not completely match either part of the second concept.

Third, consider an and concept with or instances.

Concept:

Black Unbordered object

a correct instance:

pair or large or red or

unbordered object

an incorrect instance:

pair or large or red unbordered

object.

Explanation:

One or instance part (disjunct) must be completely matched by the and concept for the instance to be correct. (Note that instance and concept are reversed from the previous rule.) Thus in the above correct instance "unbordered" is completely matched by the concept "Black Unbordered." In the incorrect instance "red unbordered" is not completely matched as the concept has "Black" instead of "Red."

Fourth, consider an or concept with or instances.

Concept:

Black Bordered object or Red Unbordered object

a correct instance:

pair or small or red or bordered

objects

an incorrect instance:

pair or small or black or

bordered objects

Explanation:

There must be a complete or instance part (disjunct) matched to each of the parts (disjuncts) of the or concept for an instance to be correct. Thus in the above correct instance "red" matches "Red Unbordered" and "bordered" matches "Black Bordered." In the incorrect instance "black" and "bordered" both match the same concept part (disjunct) while there is no match for "Red Unbordered."

* * *

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